Data Association, Part II

- Associating single observations at a time often doomed.
  - Eiffel towers are <u>great</u>, but how distinctive is a 90° corner?!

- Approach
  Use multiple observations simultaneously to disambiguate!
  - One corner is weak evidence, but 6 at a time is pretty distinctive!
  - Analogy: recognizing stars vs. constellations.

- Methods (outline)
  - Maximum Likelihood data assoc.
  - JCBB (& IPJC?)
  - SCGP
  - RANSAC

ML data assoc.

Given N observations of F features, which data associations to use?

idea: if I knew the data assocs, I could solve the SLAM problem by computing the posterior distribution. This would be:

$$p(x \mid z, a)$$

state → x; obs. → z; assocs. → a

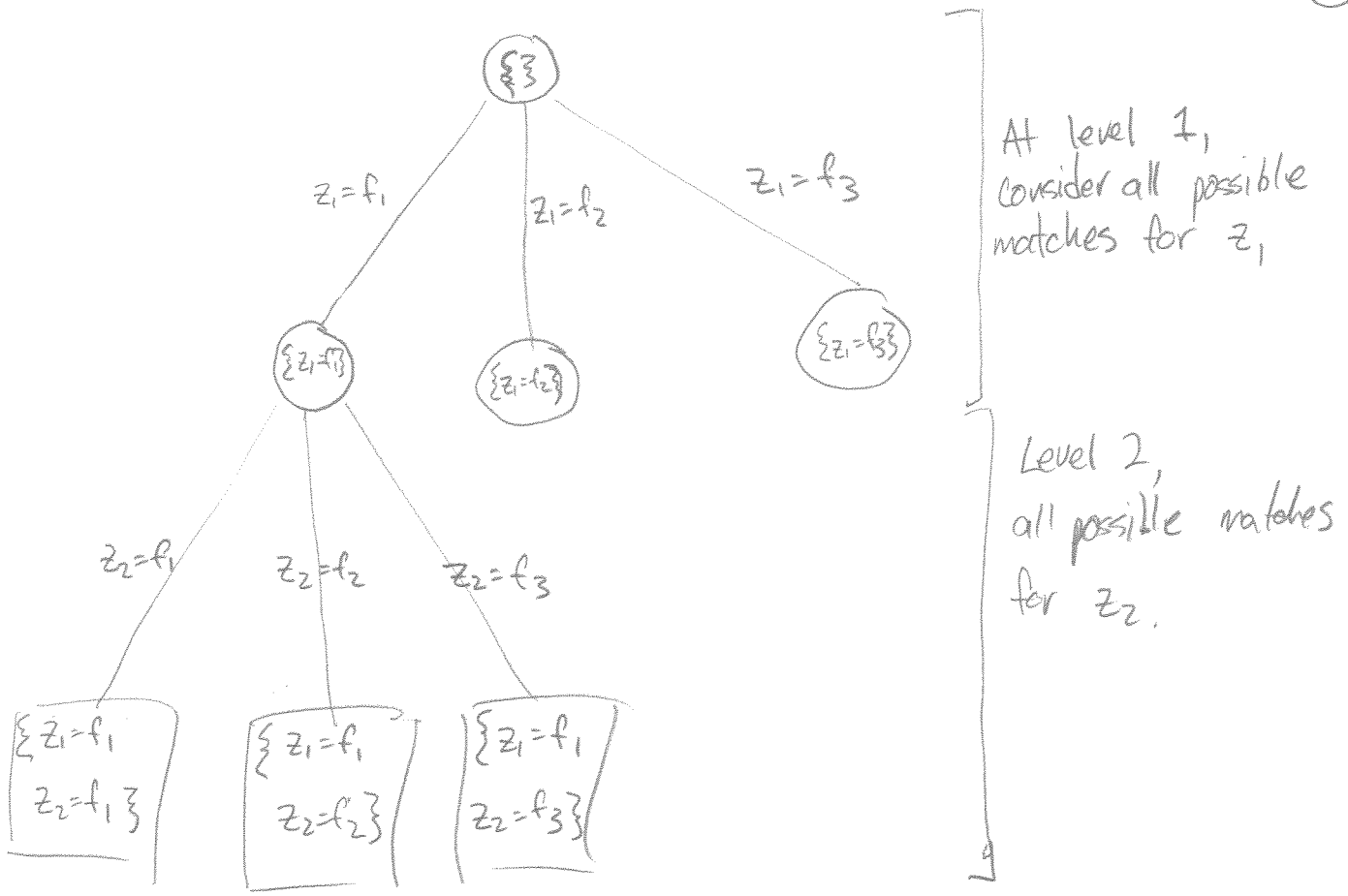
(An aside, we used to assume "a" and just write $p(x|z)$. By Bayes' rule, $p(x|z) \propto p(z|x)$, which is precisely what our least squares optimization uses as an objective function. It's important that you see this!)

An approach: Just try every value of a, compute $p(x|z,a)$ and select the a that maximizes the posterior probability of the maximum likelihood solution!

> how many values of a? F^N!

> can we search efficiently?

Yes – an "interpretation" tree



The tree terminates at level N with F^N leaves.

> At each leaf, compute posterior sol'n, (ML).

> Pick the data assoc that maximizes the p(ML sol).

(equiv., minimizes χ^2 error!)

Can we compute χ^2 at intermediate nodes?

Yes! Corresponds to ignoring observations "below".

<u>CRITICAL</u> χ^2 cost of a child always $\geq$ χ^2 cost of parent.

This yields pruning heuristic:

If you have a leaf with cost χ^2_{best}, don't expand any nodes with a greater cost.

(equiv. to A^* with a heuristic function of zero.)

How do we handle new features?

→ easy! Consider $(F+1)$ possibilities for each observation.

Uh-oh! What's the ML solution?

→ <u>every</u> observation is a new landmark!

How to fix this?

Attach a cost (or probability) to new landmarks.

$$p(z_i \text{ is new}) = K. \qquad \text{(for example.)}$$

→ This penalizes new features, "encouraging" loop closures.

→ Picking K can be hard.

JCBB

JCBB addresses the "new landmark" problem by reformulating the problem entirely!

> Instead of ML data assocs, we find the largest set of data assocs such that $\chi^2 <$ threshold.

> Branching factor still F+1, but "null" hypotheses are un-favored.

"Compatability score" same as ML, except relative to the initial χ^2, at the root of the tree.

```
besth = {{}}        ← our best data assocs,
JCBB(1, graph, obs, {});

JCBB(depth, graph, obs, h)
    if  depth = |obs|                                   (leaf?)
        if  |h| > |besth|
            besth = h
        return
    for i = 1 : F
        Cost = compute cost (graph, obs, h ∪ {Z_depth ⇒ f_i})
                                                        (see below)
        if  cost < chi2(DOF)·K
            JCBB(depth+1, graph, obs, h ∪ {Z_depth ⇒ f_i})
    JCBB(depth+1, graph, obs, h)      (handle F+1^th null hypothesis).
```

As we add more data-assocs, we expect an increase in χ^2 — even if those assocs are correct. The chi2(DOF) threshold accounts for this natural increase, DOF = # of hyperplanes resulting from associations.

JCBB optimizations

1. Do a cheap compat. test before computing the expensive test.

2. Prune if $\underbrace{|obs| - depth}_{\text{max \# of assocs that we could still make}} + \underbrace{|h|}_{\text{how many assocs. now?}} < |best\ h|$

3. Don't compute the posterior. ← The original JCBB approach.

   Instead, $$\underbrace{\underbrace{\Delta z^T \underbrace{\left(\underbrace{HPH^T}_{\text{Uncertainty in prior}} + \underbrace{GUG^T}_{\text{Uncertainty in obs}}\right)^{-1}}_{\text{Uncertainty in } \Delta z} \Delta z}_{\text{Mahl distance of } \Delta z \text{ WRT. total noise/uncertainty.}}$$

   Δz: disagreement between predicted value & obs. value

   Note: Size of this matrix grows as we descend tree
   $\Rightarrow O(depth^3)$ cost to compute compatability.

3b. Use incremental matrix inversion to reduce $O(depth^3)$ to $O(depth^2)$.

Pruning & these tricks can help a <u>lot</u>, but there are still an exponential # of leaves — can be very slow.