# First Order Logic: Conversion to CNF

1. Eliminate biconditionals and implications:
   - Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$.
   - Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$.
2. Move $\neg$ inwards:
   - $\neg(\forall x \;\; p) \equiv \exists x \;\; \neg p$,
   - $\neg(\exists x \;\; p) \equiv \forall x \;\; \neg p$,
   - $\neg(\alpha \vee \beta) \equiv \neg\alpha \wedge \neg\beta$,
   - $\neg(\alpha \wedge \beta) \equiv \neg\alpha \vee \neg\beta$,
   - $\neg\neg\alpha \equiv \alpha$.
3. Standardize variables apart by renaming them: each quantifier should use a different variable.
4. Skolemize: each existential variable is replaced by a *Skolem constant* or *Skolem function* of the enclosing universally quantified variables.
   - For instance, $\exists x \, Rich(x)$ becomes $Rich(G1)$ where $G1$ is a new Skolem constant.
   - "Everyone has a heart" $\forall x \;\; Person(x) \Rightarrow \exists y \;\; Heart(y) \wedge Has(x, y)$ becomes $\forall x \;\; Person(x) \Rightarrow Heart(H(x)) \wedge Has(x, H(x))$, where $H$ is a new symbol (Skolem function).
5. Drop universal quantifiers
   - For instance, $\forall x \;\; Person(x)$ becomes $Person(x)$.
6. Distribute $\wedge$ over $\vee$:
   - $(\alpha \wedge \beta) \vee \gamma \equiv (\alpha \vee \gamma) \wedge (\beta \vee \gamma)$.