Course Evaluations

- Thanks for the really helpful comments!
Course Evaluations: Assignments and Grading

- We’ll be providing more sample code with Psets to get you started faster. (More like Borealis, Wumpus, Elevator than Decryption).

- Increasing our grading bandwidth to get psets back to you faster.

Course Evaluations: Progress

- You now have access to your current grade information and class rank in real time:
  - [http://april.eecs.umich.edu/courses/eecs492w10/students](http://april.eecs.umich.edu/courses/eecs492w10/students)
Course Evaluations: Exams

- Lots of requests for more exam preparation!
  - In class (a la Tic-Tac Toe)
  - Sample questions and recommended R&N problems
    (watch out for this tonight!)
  - Review sessions, starting with Midterm 2.
    March 11th, 7-9p in GGBL1504
    (no prepared review; bring your questions!)

Course Evaluations: Teams

- Many comments about team assignments!
- Why do we have the system we have now?
- For PS5/PS6, you will pick your own teams.
  - Every team must have either 3 or 4 people, or be otherwise approved by the staff.
  - Send an email to eecs492-staff when you’ve formed your team.
Course Evaluations: Wrap up

- Your feedback is really helpful

- We’re trying a few new things this term
  - Thanks for your patience with the rough edges!

- End-of-term evaluations are also really important
  - But if you have additional feedback, don’t wait until then!

Course Overview: Where We Are

- Logic
  - Languages: PL, FOL
  - Inference (model checking, chaining, resolution)

- Logical Planning
  - Deterministic
  - Non-Deterministic: dealing with unknown propositions

- Probability
  - Language
  - Inference
Lecture Outline

- How does probability make for better agents?

- Decision Theory
  - What does rationality mean for a probabilistic agent?
  - Utility functions

- The language of probabilities
  - Joint, Marginal, Conditional Distributions
  - Bayes’ rule
  - Simple methods of probabilistic inference

The basics

- Probability: degree of belief
  - Pr(Intact(Tire1)) = 0.8

- What does this mean?
  - A) Tire is 80% intact
  - B) In 80% of situations like this one, the tire is intact.
  - C) This is fair to both of us: I’ll give you a dollar if you give me $1.25 if the tire is, in fact, intact.

- Fuzzy Logic: degree of truth
  - The tire is 80% intact.
  - Or maybe better: The weather is 80% miserable?
  - That’s all we have to say about fuzzy logic in this course!
Certainty through Detail

- Can approach certainty in the proposition of interest, given enough detail in causal factors.

Uncertainty and Abstraction

- Conversely, leaving out causal factors induces uncertainty in the proposition of interest.
Uncertainty as Summarization

Degrees of belief are summary measures of the uncertainty induced by leaving out model details.

Why Omit Details?

- **Ignorance**
  - Don’t have a theory of the deterministic mechanism.
  - Don’t have knowledge about the values of factors in the detailed theory. [practical ignorance]

- **Intractability (R&N “Laziness”)**
  - Impractical to specify detailed deterministic theory.
Planning under Uncertainty

- Whenever agent cannot *perfectly* predict result of executing plans
- Uncertainty may be about:
  - Initial state
  - Effect of actions
  - Exogenous events
- Examples:
  - If(WinningHand) then [BetFarm] else [Fold]
  - Intact(Tire1) ? Cannot predict whether Inflate will work

Probability

- Logic admits incomplete knowledge
  - Disjunction, negation, omission
  - But cannot express degrees of belief
  - Bird(x) -> Flies(x) ???
  - What if there’s no plan that can *always* be proven to work? How do we make our system behave as reliably as possible?

- Probability *summarizes* uncertain belief state
  \[
  \Pr(\ \text{Intact}(\text{Tire1}) ) = 0.8 \\
  \Pr(\ \text{Bird}(x) \rightarrow \text{Flies}(x)) = 0.95
  \]
KBs?

- Probabilities represent information
  - Worst-case: uniform distribution
- If I learn something:
  - Probabilities can change

- Contrast with Logic-based KB
  - Worst-case: proposition unknown
  - If I learn something: can entail new propositions

Decision under Uncertainty

Pr( Intact(Tire1) )

| 0       | Replace | ??? | Inflated | 1       |

- If intact sufficiently likely, should inflate, otherwise replace
- Threshold depends on:
  - Cost of inflating
  - Cost of replacing with spare
Decision Theory

- Represent uncertain beliefs using probability
- Represent preferences for possible outcomes using utility
- Select action (plan) maximizing expected utility

*A rational probabilistic agent maximizes expected utility*

Probability Theory

- Probability function
  \[ Pr: S \rightarrow [0,1] \]
  - \( S \) is set of sentences in a logic (typically propositional)
- Random variables (boolean, discrete, continuous)
  - Analogous to a propositional symbol
- Axioms
  1. \( 0 \leq Pr(a) \leq 1 \)
  2. \( Pr(\text{true}) = 1 \) and \( Pr(\text{false}) = 0 \)
  3. \( Pr(a \lor b) = Pr(a) + Pr(b) - Pr(a \land b) \)
Your turn!

Given:
\[ \Pr(a \lor b) = \Pr(a) + \Pr(b) - \Pr(a \land b) \]

Show that:
\[ \Pr(\neg a) = 1 - \Pr(a) \quad \text{Hint: Consider } \Pr(a \lor \neg a) \]

Justifying the Axioms

- Axioms of probability restrict the set of probabilistic beliefs an agent can hold

- Why are these beliefs irrational?
  - \( \Pr(a) = 0.4, \Pr(b) = 0.3, \Pr(a \lor b) = 0.8 \)

- De Finetti’s argument
  - Agent should be willing to bet based on beliefs
  - If \( \Pr(a) = 0.4 \), then agent should be indifferent to
    \[ [ \text{$6 if } a; \text{$4 if } \neg a ] \]
  - Any agent violating axioms can be turned into a money machine (!) via a Dutch Book
Dutch Book Example

<table>
<thead>
<tr>
<th>Event</th>
<th>Belief</th>
<th>Odds</th>
<th>Agent2 bets on</th>
<th>Outcome for Agent1</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.4</td>
<td>4 : 6</td>
<td>a</td>
<td>-6</td>
</tr>
<tr>
<td>b</td>
<td>0.3</td>
<td>3 : 7</td>
<td>b</td>
<td>-7</td>
</tr>
<tr>
<td>a ∨ b</td>
<td>0.8</td>
<td>2 : 8</td>
<td>¬(a ∨ b)</td>
<td>2</td>
</tr>
</tbody>
</table>

Joint Probability

- Probability of multiple propositions, considered simultaneously.
  - \( P(H_1 \land \neg H_2) = 0.25 \) (joint probability)

- Specifying joint probability over all atomic events = full joint distribution = complete probabilistic description of the world
  - In discrete case, could use a big table
  - How many entries in the table?
    - Assume \( N \) binary random variables

Crooked coin \( (P=0.7) \)

<table>
<thead>
<tr>
<th></th>
<th>H2</th>
<th>~H2</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1</td>
<td>0.49</td>
<td>0.21</td>
</tr>
<tr>
<td>~H1</td>
<td>0.21</td>
<td>0.09</td>
</tr>
</tbody>
</table>
Marginal Probability

- Start with a joint probability, ignore some random variables

- Crooked coin
  - Given full joint distribution
  - Suppose we can’t see the second flip
    - Can we still characterize P(H1)?

\[ P(H1) = P(H1, H2) + P(H1, \sim H2) = 0.7 \] (whew!)

Conditional Probability

\[ \Pr(a \mid b) = \frac{\Pr(a \land b)}{\Pr(b)} \]

- Undefined if \(\Pr(b) = 0\)
- Means probability of \(a\) given all we know is \(b\)

- Often: \(P(a \mid KB)\)
Your turn: Marginals

<table>
<thead>
<tr>
<th>#Legs</th>
<th>Species</th>
<th>$P(\text{Legs}=#\text{Legs}, \text{Species}=\text{Species})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Dog</td>
<td>.001</td>
</tr>
<tr>
<td></td>
<td>Cat</td>
<td>.001</td>
</tr>
<tr>
<td></td>
<td>Bird</td>
<td>.2</td>
</tr>
<tr>
<td>3</td>
<td>Dog</td>
<td>.057</td>
</tr>
<tr>
<td></td>
<td>Cat</td>
<td>.04</td>
</tr>
<tr>
<td></td>
<td>Bird</td>
<td>.001</td>
</tr>
<tr>
<td>4</td>
<td>Dog</td>
<td>.4</td>
</tr>
<tr>
<td></td>
<td>Cat</td>
<td>.3</td>
</tr>
<tr>
<td></td>
<td>Bird</td>
<td>0</td>
</tr>
</tbody>
</table>

1. $P(#\text{legs}=2 \lor #\text{legs}=3 \lor #\text{legs}=4)$
2. $P(\text{Dog} \lor \text{Cat} \lor \text{Bird})$
3. $P(\text{Bird})$
4. $P(\text{Bird}, #\text{legs}=2)$
5. $P(\text{Bird} \mid #\text{legs}=2)$
6. $P(#\text{legs}=3 \mid \text{Cat})$

PS2 Challenge Winners
PS2 Implementation

**Pre-processing steps:**
- Input sentence parsed and broken into individual words
- Heuristic used to conservatively eliminate proper nouns from the search
- Domain of each variable initially constrained to words in the dictionary of the same length matching the same letter pattern

<table>
<thead>
<tr>
<th>Initial Domain</th>
<th>Encrypted Word</th>
<th>Final Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABASH, ARRAY, AVAIL, ...</td>
<td>ABACD</td>
<td>ABASH, AVAIL, ...</td>
</tr>
</tbody>
</table>

**Search:**
- Select the variable with the minimum remaining values in its domain for next assignment
- Constrain the domains of all remaining unassigned variables
- Repeat the above, continuing a depth-first search of all possible assignments

**Other considerations:**
- If the search completed without finding a solution, additional potential proper nouns are removed and the search is repeated

**Optimizations:**
- Order in which dictionary words are tried during the search is heuristically determined based upon the trigram frequency count of each word in comparison to its known frequency in English text.
  - [http://home.ccil.org/~cowan/trigrams](http://home.ccil.org/~cowan/trigrams)
- Data structures chosen to best facilitate domain computation for each variable during the search
Evidence Evaluation Example

- **Disease testing (hypothetical):**
  - Prior probability (prevalence)
    - $\Pr(\text{disease}) = .0005$
  - Conditions (test accuracy)
    - $\Pr(\text{pos test} | \text{disease}) = 1$
    - $\Pr(\text{neg test} | \neg \text{disease}) = .995$
  - Posterior
    - $\Pr(\text{disease} | \text{pos test}) = ?$

Product Rule

- $P(A | B) P(B) = P(A, B)$

- Of course $P(A, B) = P(B, A)$, so:
  - $P(A | B) P(B) = P(B | A) P(A) = P(A, B)$

- If we rearrange a bit, we arrive at one of the most important probabilistic theorems:
Bayes’s Theorem

\[
Pr(h \mid e) = \frac{Pr(h \land e)}{Pr(e)}
\]

\[
= \frac{Pr(e \mid h)Pr(h)}{Pr(e)}
\]

Evidence Evaluation Example

- **Disease testing (hypothetical):**
  - **Prior probability (prevalence)**
    - \( Pr(\text{disease}) = .0005 \)
  - **Conditionals (test accuracy)**
    - \( Pr(\text{pos test} \mid \text{disease}) = 1 \)
    - \( Pr(\text{neg test} \mid \neg \text{disease}) = .995 \)
  - **Posterior**
    - \( Pr(\text{disease} \mid \text{pos test}) = \)
      \[
      \frac{Pr(\text{pos test} \mid \text{disease}) Pr(\text{disease})}{Pr(\text{pos test})} = \frac{1 * .0005}{??}
      \]
Evidence Evaluation Example

Pr(pos test) = Pr(pos-test ^ disease) + Pr(pos-test ^ ~disease)

= Pr(pos-test | disease)Pr(disease) + Pr(pos-test | ~disease)Pr(~disease)

= (1 * .0005) + ((1-Pr(neg|~dis))*(1-Pr(disease)))

= .0005 + (.005 * .9995)

= .0054975

Evidence Evaluation Example

- Disease testing (hypothetical):
  - Prior probability (prevalence)
    - Pr(disease) = .0005
  - Conditionals (test accuracy)
    - Pr(pos test | disease) = 1
    - Pr(neg test | ¬ disease) = .995
  - Posterior
    - Pr(disease | pos test) =
      Pr(pos test | disease) Pr(disease) / Pr(pos test) =
      1 * .0005 / .0054975 =
      0.09095
Causal versus Diagnostic Information

- $P(\text{funny engine noise} \mid \text{loose hose})$
  - Causal or Diagnostic?
- $P(\text{loose hose} \mid \text{funny engine noise})$
  - Causal or Diagnostic?

Bayes’ rule allows us to go back and forth
- Which fact is more useful?
- News report: “Police have identified the notorious hose loosener, who has doubled the prevalence of loose hoses. This hose loosener is still on the loose!”
  - What is $P(\text{funny engine noise} \mid \text{loose hose})$ now?
  - What is $P(\text{loose hose} \mid \text{funny engine noise})$ now?

Independence

- $a$ and $b$ are independent iff:
  - $Pr(a \mid b) = Pr(a)$
- Independence implies
  - $Pr(a \land b) = Pr(a)Pr(b)$
- $a$ and $b$ are conditionally independent given $c$ iff:
  - $Pr(a \mid b \mid c) = Pr(a \mid c)$
  - Equiv: $P(a, b \mid c) = P(a \mid c)P(b \mid c)$
Combining Conditions

- How to calculate
  - \( \Pr( \text{Intact} | \text{Flat}, \text{Glass}) \)

- Given
  - \( \Pr( \text{Flat} | \text{Intact} ), \Pr( \text{Flat} | \sim \text{Intact}) \)
  - \( \Pr( \text{Intact} | \text{Glass} ), \Pr(\sim \text{Intact} | \text{Glass}) \)
  - Flat (looks flat) is conditionally independent of Glass (glass in road) given Intact

- **Hint**: use the conditional independence, normalization

Continuous-valued Probabilities

- So far, we’ve only described discrete-valued probabilities
- Many real-world quantities are continuous
  - Tire pressure
  - GPS coordinates of car

- Very similar to discrete-valued probabilities...
Continuous-valued Probabilities

<table>
<thead>
<tr>
<th>Discrete</th>
<th>Continuous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability functions</td>
<td>Probability Density Functions</td>
</tr>
<tr>
<td>$P(S) = [0, 1]$</td>
<td>$P(x) \geq 0$</td>
</tr>
<tr>
<td>$\sum P(S) = 1$</td>
<td>$\int P(x) , dx = 1$</td>
</tr>
<tr>
<td>$\text{Prob}(x) = 0$</td>
<td>$\text{Prob}(x) = 0$</td>
</tr>
</tbody>
</table>

• Despite differences, notation for discrete probability distribution and continuous probability density function is (usually) the same!

• Common continuous distributions
  • Uniform: $U(0,5)$
  • Gaussian: $N(\mu, \sigma^2)$

Philosophy: Where do probabilities come from?

- **Frequentist**
  - Probabilities computed from empirical data

- **Objectivist** [real propensities]
  - Probabilities are a *real property* of the universe

- **Subjectivist**
  - Probabilities characterize an agent’s beliefs.

- *Issues*: unique events, reference classes, measurement, ...
Next Time

- Bayesian Networks
  - Full joint distributions can be very big
  - The world has structure: not every proposition is correlated with very other proposition!
  - Exploit conditional independence to reduce problem size
  - Faster inference