Kalman Filter related equations

Let the state vector be $s = \begin{bmatrix} x \\ y \\ v_x \\ v_y \end{bmatrix}$

The state at the next time step is $s^{'} = \begin{bmatrix} x + v_x dt \\ y + v_y dt \\ v_x + w_1 \\ v_y + w_2 \end{bmatrix}$

This is of the form $s^{'} = f(s, w)$ where $w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$.

Time Propagation:
Let $s0$ denote the state in the previous time step and s^{-} the estimate of the current state based on the time propagation only. This estimate s^{-} is also called apriori estimate of the state.

- state estimate using time propagation:
  $s^{-} = f(s0, 0)$.

- covariance estimate:
  $\Sigma_s^{-} = J_s^f \Sigma_s^f J_s^{fT} + J_w^f \Sigma_w^f J_w^{fT}$

Jacobian calculations:

- $J_s^f(i,j) = \frac{\partial f_i}{\partial s_j}(s0, 0)$
- $J_w^f(i,j) = \frac{\partial f_i}{\partial w_j}(s0, 0)$

Observation:
Our observation vector is the following
$z = \begin{bmatrix} r + v_1 \\ \theta + v_2 \end{bmatrix} = \begin{bmatrix} \sqrt{(x - x_a)^2 + (y - y_a)^2} + v_1 \\ \arctan \frac{y - y_a}{x - x_a} + v_2 \end{bmatrix}$
This is of the form $z = h(s, v)$. Here $v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$, $v_1 \sim N(0, \sigma_r^2)$, $v_2 \sim N(0, \sigma_\theta^2)$ and (x_a, y_a) is the position of your agent.

Kalman filtering (Incorporating the observation with the propagation)

- Kalman gain:
  $K = \Sigma_s^{-} J_s^{hT} (J_s^h \Sigma_s^{-} J_s^{hT} + J_v^h \Sigma_v^h J_v^{hT})^{-1}$
- The posteriori estimate
  $s = s^{-} + K(z - h(s^{-}, 0))$
- The updated covariance
  $\Sigma_s = \Sigma_s^{-} - K J_s^h \Sigma_s^{-}$

Jacobian calculations:

- $J_s^h(i,j) = \frac{\partial h_i}{\partial s_j}(s^-, 0)$
- $J_v^h(i,j) = \frac{\partial h_i}{\partial v_j}(s^-, 0)$

For the next iteration you will use s and the covariance, obtained above through Kalman filtering, as $s0$ and Σ_s^f in your time propagation step.

Note: Throughout all the expressions 'T' in the superscript means transpose.