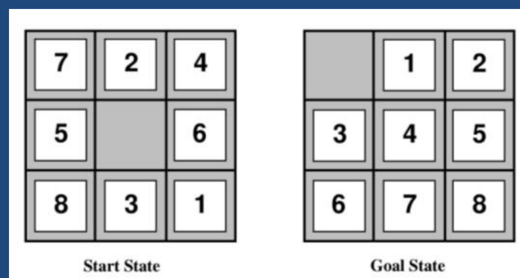


Informed Search

EECS 492
January 19th, 2010

Administrative

- PS1 due Thursday!
- My office hours moved to today

Last Time: Uniformed Search

- General-Purpose
  - Require only the problem definition itself:
    - *state*$_0$
    - successors(*state*)
    - is-goal(*state*)
    - cost(path)
- Powerful
  - Several Complete and Optimal algorithms to choose from!

Analysis Summary

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes	Yes	No	No	Yes
Optimal?	Yes	Yes	No	No	Yes
Time complexity	$O(b^{d+1})$	$O(b^{C^*/\varepsilon+1})$	$O(b^m)$	$O(b^L)$	$\boxed{O(b^d)}$
Space compexity	$O(b^{d+1})$	$O(b^{C^*/\varepsilon+1})$	$O(bm)$	$O(bL)$	$\boxed{O(bd)}$

Exponential in depth!

Linear in depth!

Uninformed Search

- All we have is:
  - problem = {$state_0$, is-goal(), cost(), successors() }
- We have no idea how well we're doing until we suddenly find a goal!

Informed Search

- We still need to know the problem
  - problem = {$state_0$, is-goal(), cost(), successors() }
- We get some additional information at each node
  - "Hot" or "Cold"
  - Which states are better than others?
  - Information about distance to goal
  - Formulate as a real-valued metric: f(*node*).

Informed Search

- Exploit additional information of f() such that:
  - We find the goal faster when f() is accurate
- With the right algorithm:
  - We still (eventually) find a goal when f() is wrong
  - We can find the *optimal* goal if f() is only wrong in certain ways!
    - Generally easy to satisfy conditions too!

Best-First Search

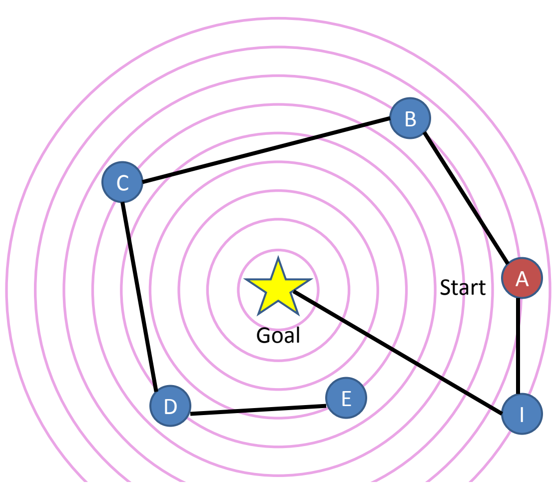
- It's really just TreeSearch again, except we allow more types of Queue-Get functions
- For each node, define an *evaluation function* f(node).
  - Having f() is how we "inform" the search!
  - Queue-Get: Expand the node with the smallest value of f()
- Watch out for "BFS"
  - Breadth-first search or best-first search?

Greedy Search

- Simple informed search with f() = cost-to-go(n)
- Complete?
- Optimal?

Greedy Search: Example

- Queue-Get: returns the node with minimum f(n)=cost-to-go(n)
- Assume we avoid repeated states.



A (8)

Expand A:
AB (7)
AI (9)

Expand AB:
ABC (6)
AI (9)

Expand ABC:
ABCD (5)
AI (9)

Expand ABCD:
ABCDE (4)
AI (9)

Expand ABCDE:
(dead end)
AI (9)

Expand AI:
AI*

A*

It's a Best-First Search!

Thrilling. Yet Another TreeSearch. What's Queue-Get this time?

Glad you asked! Queue-Get returns the node with
the minimum value of:

f(n) = cost-so-far(n) + h(n),
where h(n) has some special properties.

What's it good for?

A*

- Provided the heuristic is *admissible*:
  - A* is complete
  - A* is optimal
  - A* is *optimally efficient*

Optimally Efficient?

No algorithm can expand fewer nodes than A* and still be guaranteed to find the optimal answer.

A*: Admissible heuristics

- Admissible:

h(n) <= the minimum achievable cost from n to the goal.

A*: Proof of Optimality

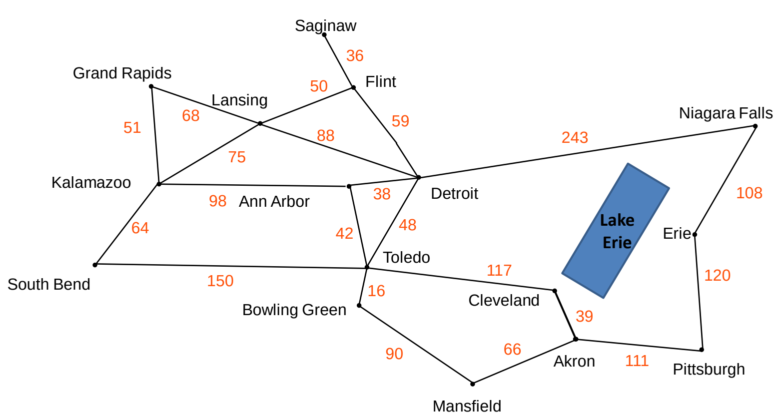
- Strategy:
  - Let C* be cost of optimal solution
  - Show that a node on the path to the optimal solution will always be selected for expansion before a sub-optimal goal node.
    - Eventually, that node will be the goal node and we'll be done.

- Proof
  - Suppose a suboptimal goal *G'* in a node on fringe
    - G' is suboptimal ←→ g(G') > C*
    - f(G')=g(G')+h(G')
    - f(G') > C*.
  - There must be a node *n* on fringe that is on optimal path, and because *h(n)* can't overestimate, *f(n) < C**
  - Since *f(n) < f(G'),* we'll expand n before G'.

Admissible Heuristics

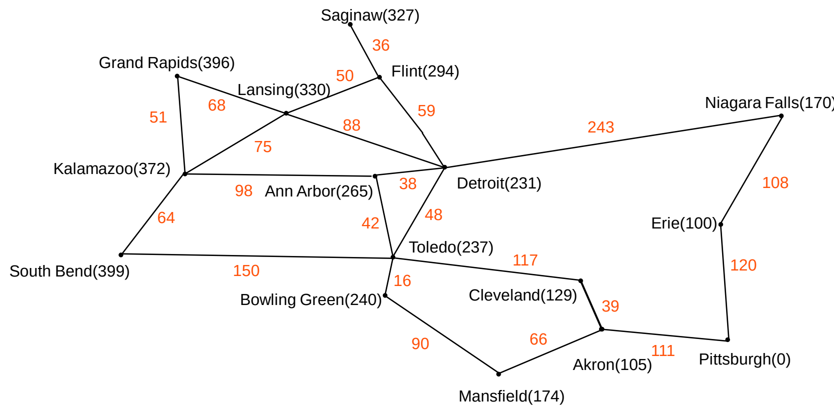
- Objectives:
  1. Accurate estimate of distance to goal
  2. Never overestimate (admissible)
  3. Easy to compute
- Approaches:
  - Just think of something
  - Relax constraints
  - Learn from experience

A*: Example



What's a good heuristic?

A*: Example



A* Example: Solution

1: {AA(265)}

3: { **AA-D(269), AA-T(279), AA-K(470)**}

7: { AA-T(279), **AA-D-T(323), AA-D-AA(341), AA-D-F(391), AA-D-N(451), AA-D-L(456),**AA-K(470) }

11: { **AA-T-C(288), AA-T-B(298), AA-T-D(321),** AA-D-T(323), AA-D-AA(341), **AA-T-AA(349)**, AA-D-F(391), AA-D-N(451), AA-D-L(456), AA-K(470), **AA-T-SB(591)** }

12: { AA-T-B(298), **AA-T-C-A(303)**, AA-T-D(321), AA-D-T(323), AA-D-AA(341), AA-T-AA(349), AA-D-F(391), AA-D-N(451), AA-D-L(456),AA-K(470), **AA-T-C-T(513)**, AA-T-SB(591) }

13: { AA-T-C-A(303), **AA-T-B-T(311)**, AA-T-D(321), **AA-T-B-M(322)**, AA-D-T(323), AA-D-AA(341), AA-T-AA(349), AA-D-F(391), AA-D-N(451), AA-D-L(456),AA-K(470), AA-T-C-T(513), AA-T-SB(591) }

15: { **AA-T-C-A-P(309)**, AA-T-B-T(311), AA-T-D(321), AA-T-B-M(322), AA-D-T(323), **AA-T-C-A-C(366)**, AA-D-AA(341), AA-T-AA(349), AA-D-F(391), **AA-T-C-A-M(444) ,** AA-D-N(451), AA-D-L(456), AA-K(470), AA-T-C-T(513), AA-T-SB(591) }

Your turn!

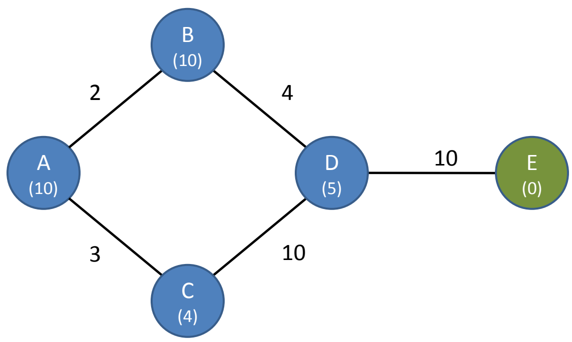
- 1. Which algorithm do we get if we do Best First Search (BFS) with:
  - A. f() = num-actions(node)
  - B. f() = cost-so-far(node)
  - C. f() = cost-to-go(node)
  - D. f() = cost-so-far(node) + cost-to-go(node)
  - E. f() = time-in-fringe(node)
  - F. f() = -time-in-fringe(node)

Graph Search and A*

- Tree search A* has same problem as other tree searches... (what problem is it?)
  - Can re-expand same states many times over.
- Graph Search was the answer…
  - Don't add paths to the fringe when we already know how to get to that state.

Graph Search A*

- Use GraphSearch/A*



A (10)

AB (12)
AC (7)

AB (12)
~~ACA (16)~~
ACD (18)

~~ABA (14)~~
~~ABD (11)~~
ACD (18)

ACDE (23)

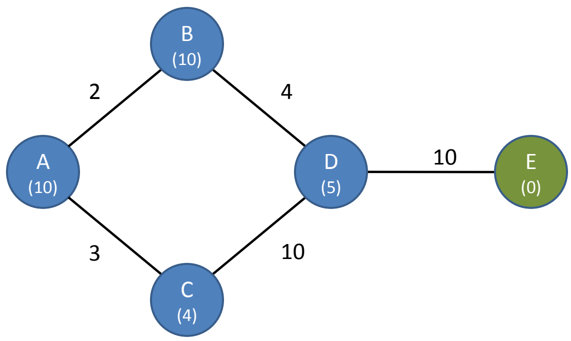
Sub-optimal!
(why did this happen?)

A stronger heuristic: Consistency

- Admissibility
  - h(n) <= true cost from n to goal
- Consistency (Monotonicity)
  - h(n) <= c(n, n') + h(n')
  - (Implies admissibility. Why?)
    - h(n) <= c(n, n') + [c(n', n'') + h(n'')]
- A* optimal if:
  - Tree: h() is admissible
  - Graph search: h() is consistent

Consistency

- Where is this function non-consistent?
  - Consistency property: h(n) <= c(n, a, n') + h(n')



Consistency and Tree Search

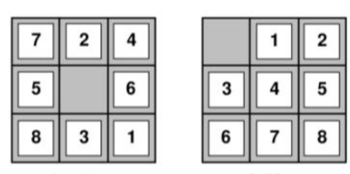
- Why *don't* we need consistency for tree search?

Constructing Admissible Heuristics

- Relax constraints
- Sub-problems
- Pattern databases

Generating Heuristics by Relaxation

Calculate exact distance for *relaxed* version of problem



Allow tile to be "teleported" to destination

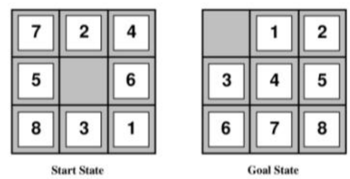
h1: #misplaced tiles

Allow move to adjacent square even if occupied

h2: Manhattan distance

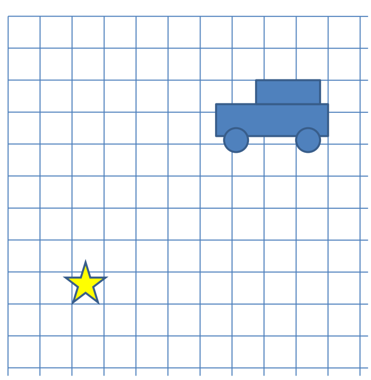
Generating heuristics with sub-problems

- $h_3(n)$ = How many moves to get tiles 1-4 into the correct position?
- $h_4(n)$ = How many moves to get tiles 5-8 into the correct position?



Pattern Database

- Stanford Parking Planner
  - Precompute distance to adjacent cells assuming no obstacles



Admissible Heuristics

- Which heuristic is better?
  - The one that produces the larger values.

- Are there admissible h_1, h_2 such that
  $h_1(n_i) > h_2(n_i)$ and $h_1(n_j) < h_2(n_j)$?

- Domination
  - $h_1(n) >= h_2(n)$ for all n.

Combining Multiple Heuristics

- Suppose we have multiple admissible heuristics.
  - What is the optimal combination of them?

- What if the heuristics are disjoint?
  - I.e., progress on one heuristic can not affect progress of another heuristic

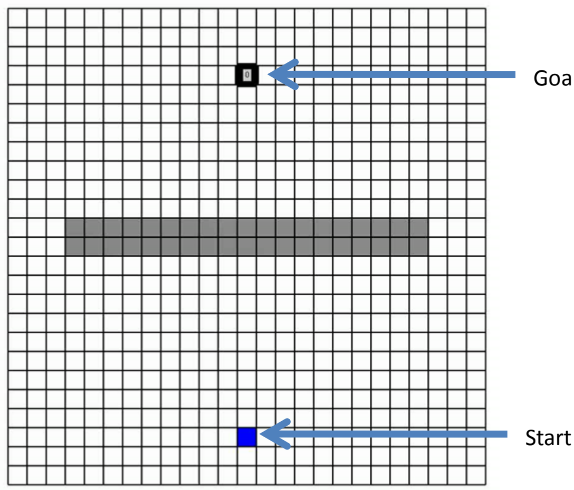
Inadmissible Heuristics

- Learning heuristics based on features
  - $H(n) = Ax(n) + By(n)$
  - Where A,B are parameters fit to observed data.
- Are these useful?

A*: Not a panacea

- Avoiding exponential search size requires heuristic error to grow no faster than log(cost)
- This is hard to do: error often proportional to cost.
  - Consider: Straight-line distance?
- Otherwise, memory/CPU are $O(b^n)$
  - Memory will generally be the limiting problem
  - Sound familiar?

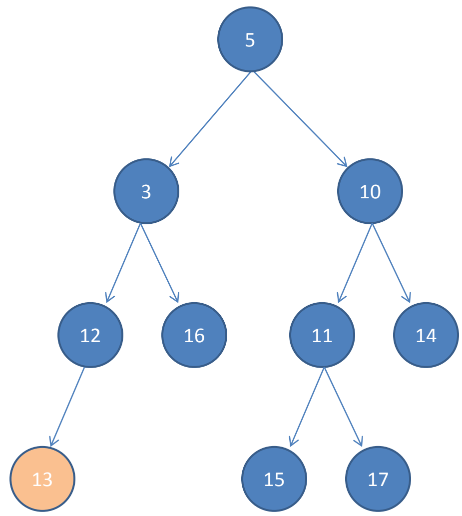
A*



Recursive Best First Search (RBFS)

- RBFS is a scheme to reduce the memory requirements
- Each node knows the f() of the best alternative path from one of its ancestors
  - If the current f() value exceeds this limit, we unwind back to the common ancestor, then re-expand the tree down the alternate path.
- Two (or more) paths can wrestle control back and forth:
  - Constantly re-expanding nodes
- Space:
  - O(d)

RBFS Example



IDA*

- IDA* is a scheme to reduce the memory requirements
- Virtually identical to IDS:
  - Instead of Depth-Limited Search, use f()-limited search
  - Upon failure, f()-limit is increased to the lowest f() value that was previously pruned.
- Time:
  - $O(b^{C^*/e})$
- Space:
  - O(bd)

SMA*

- RBFS and IDA* suffer from using too *little* memory.

- Idea: Remember as much of the search tree as we can. (Delay the onset of thrashing as long as we can!)

  - 1. When the queue is not too big
    - Expand the leaf in the search tree with the minimum f()
  - 2. When memory runs out:
    - Find the leaf in the search tree whose f() is the worst and remove it.
    - Back up its f() to its parent. Note that the parent might now become a leaf

Next time

- Local Search