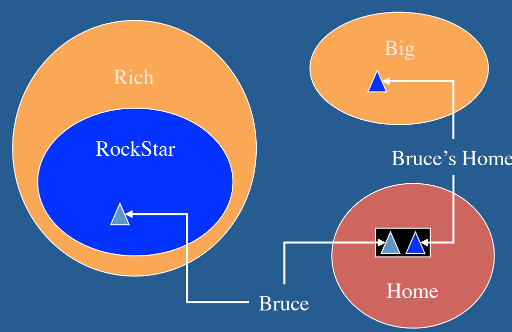
$\forall x\ \exists y.\ \Phi(x,y)$



FIRST-ORDER LOGIC

EECS 492
February 10th, 2011

Propositional Logic

- Advantages
  - Simple
  - Generic
- Disadvantage
  - All we have are propositions (facts)
  - Q: What else would we want?

PL for Vacuum World

- Let D_i, i = 1,…,n, represent "room i is dirty"
- How to say the following?
  - All the rooms are clean
  - The dog is in some room
  - Any room that the dog has been in is dirty
  - All the upstairs rooms are clean, except the bathrooms
  - Rooms next to dirty rooms have a dusty smell
  - No two adjacent rooms are both dirty
  - The vacuum should visit every dirty room
  - When vacuuming a room, check whether any nearby rooms are dirty

PL can be Cumbersome

- For example, introducing *NextTo* required O(n^2) new propositional symbols
- Inference exponential in number of propositional symbols
  - Infinite domains… forget about it
- Solution: put more *structure* on facts

First Order Logic

- Create a richer language by adding additional structure.

Propositional Logic	First Order Logic
Logically reason over *propositions*	Logically reason about *objects*, their properties, and their relationships. A superset of PL.

First Order Logic

- Objects
  - Rooms, dogs, vacuums, wumpus rooms…
- Predicates
  - Tests a property of one or more objects
    - IsBreezy(x), AreAdjacent(x,y), isCleanerThan(x, y)
  - Value (true or false) can be evaluated given a model
  - With no arguments, equivalent to PL propositions.

Arity??

- Functions
  - Names an *object* as it relates to other objects
    - MotherOf(x), RoomEastOf(x), ChildOf(x,y)
  - By themselves, do not form legal sentences; used along with predicates.
  - With no arguments, is a *constant object*…. E.g., "John".

Predicates and Functions

- These look like "function calls" from Java or C, but are *quite* different!
- Defined implicitly, not explicitly.
  - Consider:
    - Male(John) ^ Male(Arnold)
    - AreBrothers(John, Arnold) v AreCousins(John, Arnold)
    - FatherOf(John) = FatherOf(Arnold)
  - Relationships are built up from sentences of FOL, in which relationships may appear anywhere.
    - The properties are a matter of *inference*.

Syntax of First-Order Logic

Sentence	→	AtomicSentence | ¬Sentence
		| (Sentence Connective Sentence)
		| Quantifier Variable, … Sentence
AtomicSentence	→	Predicate(Term,…) | Term = Term
Term	→	Function(Term,…) | Constant | Variable
Connective	→	⇒ | ∧ | ∨ | ⇔
Quantifier	→	∀ | ∃
Constant	→	*A* | X_1 | Room1500EECS | …
Variable	→	*a* | *x* | *s* | …
Predicate	→	*IsDirty* | *IsNextTo* | *IsComfy* | …
Function	→	*RoomOnRight* | *FurnitureInRoom* | …

Equality

- Special built-in predicate symbol
- *Term* = *Term*
  - An atomic sentence
  - =: a binary predicate, equivalence reln
  - Written using infix notation
- Similarly, !=.

FOL Models

- Just like in PL, we can evaluate the *satisfiability* of a sentence *with respect to a model.*
- What was a model in PL?
  - Just an assignment of true/false to every proposition
- What is a FOL Model?

FOL Models

- A model must specify:
  - Which constants represent the same objects
    - IsCat(Felix) ^ IsDog(Fido)
    - Owner(Felix) = Robert
    - Owner(Fido) = Mr.Haroldson
    - HomeOf(Felix) = HomeOf(Fido)
    - LivesAlone(Robert)
  - Behavior of predicates
    - A truth table over all possible inputs
  - Behavior of functions
    - A map over all possible inputs

A Review Game

Making FOL even more powerful

- So far:
  - A neater way to write propositions
  - Captures object/relation structure, but still need to enumerate all cases
- Real FOL power comes from ability to quantify over objects using variables

Universal Quantifier

- "All the rooms are clean"
  - $\forall x.\ \neg Dirty(x)$
  - $\forall x.\ Room(x) \Rightarrow \neg Dirty(x)$

variable

universal quantifier

Equivalent to big conjunction, where x is replaced by every object in universe

Existential Quantifier

- "The dog is in some room"
  - $\exists x.\ DogIn(x)$
  - $\exists x.\ Room(x) \land DogIn(x)$

existential quantifier

Equivalent to big *dis*junction, where *x* is replaced by every object in universe

"First-Order"

- First-order logic (FOL)
  - allows quantification over *objects*
- Second-order logic
  - allows quantification over *relations*
  - can describe properties of relations (e.g., transitivity)
- Higher-order logic (HOL)
  - Quantify over relations over relations…
- Zeroth-order logic
  - No quantification
  - (aka propositional logic)

Sentences with Variables

- $\forall x.\ \Phi(x)$
  - True in model iff $\Phi(x)$ true *no matter what* object x denotes
- $\forall x.\ Room(x) \Rightarrow \neg Dirty(x)$
  - True iff $Room(x) \Rightarrow \neg Dirty(x)$ is true for all objects x
  - Trivially true for any x not a room
- $\exists x.\ Room(x) \wedge DogIn(x)$
  - True iff there is some x s.t. x is a room with a dog in it

- Compare and contrast:
  - $\exists x.\ Room(x) \Rightarrow DogIn(x)$
  - $\forall x.\ Room(x) \wedge \neg Dirty(x)$

Sentences about Rock Stars' Houses

- Rock stars are rich.
- Everybody has a home.
- Rich people have big homes.
- Bruce is a rock star.

Rock Star House KB

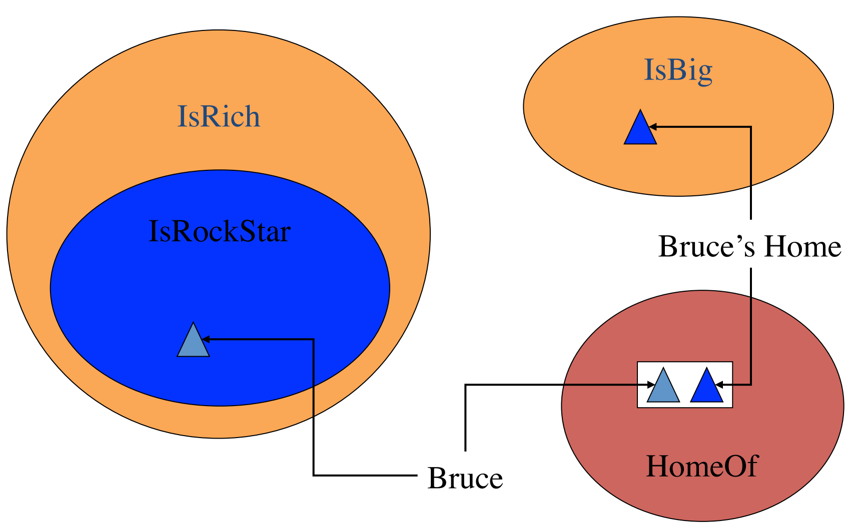
$\forall p.\ IsRockStar(p) \Rightarrow IsRich(p)$

$\forall p.\ IsPerson(p) \Rightarrow \exists h.\ HomeOf(h,p)$

$\forall p.\ IsRich(p) \Rightarrow \exists h.\ HomeOf(h,p) \wedge IsBig(h)$

IsRockStar(Bruce)

Rock Star House Model

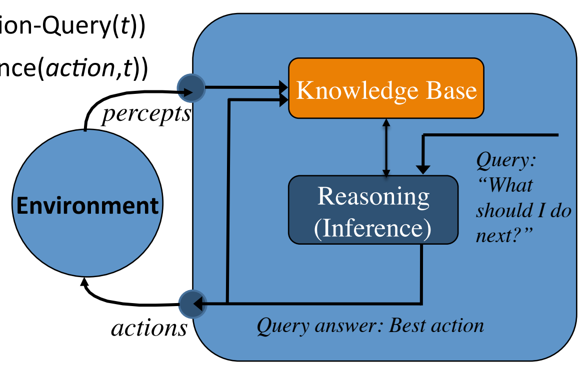


Declarative Approach

- Program an agent by TELLing it things
- Equivalently, construct and install a KB
- Advantages
  - Flexibility: state knowledge independently of how it would be used
  - Transparency to humans
  - Agent behavior *malleable via language*

Knowledge-Based Agent

function KB-Agent(*percept*) **returns** an *action*
  static: a knowledge base, *KB*
    a counter, *t*, indicating time
  Tell(*KB*,Make-Percept-Sentence(*percept*,*t*))
  action ← Ask(*KB*,Make-Action-Query(*t*))
  Tell(*KB*,Make-Action-Sentence(*action*,*t*))
  t ← *t*+1
  return *action*



Developing a KB Agent

- Endow with KB axiomatizing domain
  - Initial situation
  - Causal laws (how world evolves)
  - Etc.
- Develop techniques for
  - Describing percepts as sentences
    - E.g., observe "facts" that hold
  - Get action as result of ASK

Reflex KB Agent

- Actions determined by rules in reaction to situation

$\forall x,t.\ VacuumIn(x,t) \wedge Dirty(x,t) \Rightarrow Action(Suck,t)$

- Recognizing imperfect perception:

$\forall x,t.\ VacuumIn(x,t) \wedge SeemsDirty(x,t) \Rightarrow Action(Suck,t)$

Model-Based KB Agent

- Reflex limitations
  - Cannot account for historical percepts
  - Rules easily invalidated by additional knowledge
- Model-based
  - KB describes internal model of world
  - Includes logical specification of effects of actions

$$\forall x,t.\ \text{VacuumIn}(x,t) \wedge \text{Action}(\text{Suck},t) \Rightarrow \text{Clean}(x,t+1)$$

Goal-Based KB Agents

- Goal-based: model-based, plus a rep'n of objective in terms of desirable states
- Utility-based: goal-based, but with degrees of preference
- In all these, action query implemented through logical inference.