

$$\frac{\forall v. \alpha}{\text{Subst}(\{v/g\}, \alpha)}$$

MORE FIRST-ORDER LOGIC

EECS 492
February 15, 2011

Today's goals

- Practice understanding sentences
 - ▣ FOL -> English

- Practice translating sentences
 - ▣ English -> FOL

- Start FOL inference

Symbol Names

Compare:

- ▣ $\forall x. \text{MyFriend}(x) \Rightarrow \text{SendBirthdayCard}(x)$
- ▣ $\forall y. \text{MyFriend}(y) \Rightarrow \text{SendBirthdayCard}(y)$
- ▣ $\forall x. \text{P0001}(x) \Rightarrow \text{P0002}(x)$
- ▣ $\forall x. \text{MyEnemy}(x) \Rightarrow \text{SendBirthdayCard}(x)$

FOL to English

$\forall m,c. \text{IsMotherOf}(c,m) \Leftrightarrow \text{IsFemale}(m) \wedge \text{IsParentOf}(m,c)$

$\forall w,h. \text{IsHusbandOf}(w,h) \Leftrightarrow \text{IsMale}(h) \wedge \text{IsSpouseOf}(h,w)$

$\forall x. \text{IsMale}(x) \Leftrightarrow \neg \text{IsFemale}(x)$

$\forall p,c. \text{IsParentOf}(p,c) \Leftrightarrow \text{IsChildOf}(c,p)$

$\forall g,c. \text{IsGrandparentOf}(g,c) \Leftrightarrow \exists p. \text{IsParentOf}(g,p) \wedge \text{IsParentOf}(p,c)$

$\forall x,y. \text{IsSiblingOf}(x,y) \Leftrightarrow x \neq y \wedge \exists p. \text{IsParentOf}(p,x) \wedge \text{IsParentOf}(p,y)$

Your turn:

IsGrandChildOf , $\text{IsGreatGrandparentOf}$, IsBrotherInLawOf , IsFirstCousinOf ,
 $\text{IsNthCousinOf}, \dots$

English to FOL

What's the "right" translation of the sentence "Not all students take both history and biology." ?

NotAllStudentsTakeBothHistoryAndBiology()

NotAllStudentsTakeBoth(History, Biology)

NotAllStudentsTake(History \wedge Biology)

NotAllStudentsTake(History) \wedge NotAllStudentsTake(Biology)

\neg AllStudentsTakeBoth(History,Biology)

$\neg\forall x. \text{IsStudent}(x) \Rightarrow \text{TakesBoth}(\text{History},\text{Biology})$

$\neg\forall x. \text{IsStudent}(x) \Rightarrow \text{Takes}(x,\text{History}) \wedge \text{Takes}(x,\text{Biology})$

$(\neg\forall x. \text{IsStudent}(x) \Rightarrow \text{Takes}(x,\text{History})) \wedge (\neg\forall y. \text{Student}(y) \Rightarrow \text{Takes}(y,\text{Biology}))$

More History and Biology

- Not all students take both History and Biology.
- Only one student failed History.
- Only one student failed both History and Biology.
- The best score in History was better than the best score in Biology.

Temporal Sentences

- George is the President of the United States.
- The President of the United States has lived in the White House since 1803.
- The President of the United States has been sober since 1986.

FOL Inference: Reduction to PL

- Standard PL inference rules sound for FOL as well
 - ▣ E.g., *modus ponens*

$$\frac{\alpha \Rightarrow \beta, \alpha}{\beta}$$

IsFriend(Arnold)

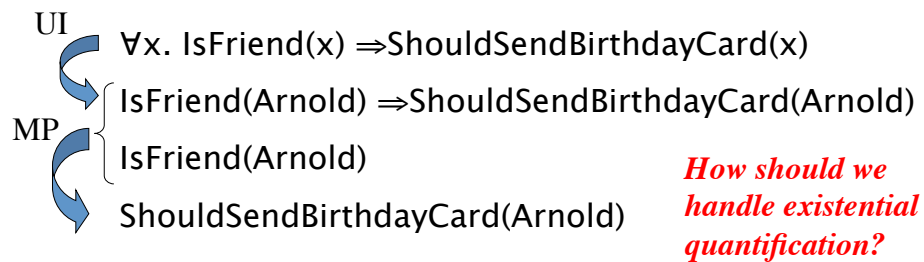
IsFriend(Arnold) \Rightarrow ShouldSendBirthdayCard(Arnold)

ShouldSendBirthdayCard(Arnold)

Universal Instantiation

- Replace a universally quantified variable with a ground term

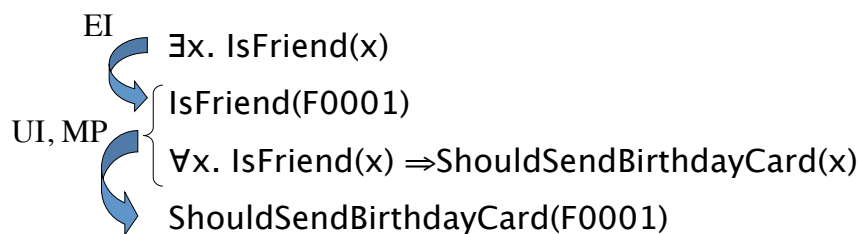
$$\frac{\forall v. \alpha}{\text{Subst}(\{v/g\}, \alpha)}$$



Existential Instantiation

- Replace an existentially quantified variable with a Skolem constant

$$\frac{\exists v. \alpha}{\text{Subst}(\{v/Sk\}, \alpha)}$$



FOL Inference

- We can now reduce FOL to PL inference:
 - ▣ Existentially instantiate everywhere.
 - ▣ Universally instantiate with respect to every object
 - ▣ Treat resulting terms as propositions
 - E.g., “IsFriend(FatherOf(Arnold))” is just a long name for a proposition.

- Uh oh!
 - ▣ Universal instantiation explodes if we have functions!
 - IsFriend(FatherOf(FatherOf(FatherOf(FatherOf(...

Herbrand's Theorem

- If a KB entails A, then there is a proof involving a *finite* subset of the propositionalized knowledge base.
 - ▣ I.e., any proof requires only a finite number of $f(f(f(\dots)))$'s

- What strategy does that suggest?

Semi-Decidability

- Does KB entail A?
 - ▣ Suppose we don't find a solution at depth 1...
 - ▣ Or depth 2.
 - ▣ Or depth 3.
 - ▣ Or depth 4.
 - ▣ ...
- When can we state that KB does NOT entail A?
- Entailment of FOL is **semidecidable**: we can prove entailments, but can't disprove every non-entailed sentence.
 - ▣ Are there some non-entailed sentences that we *can* disprove?

Another problem with FOL → PL

- We can have an infinite number of propositions!
 - ▣ Proving statements about arithmetic
- Peano Axioms
 - ▣ $\text{NatNum}(0)$
 - ▣ $\forall n. \text{NatNum}(n) \Rightarrow \text{NatNum}(S(n))$

Today's big idea

- We can reduce FOL to PL
 - ▣ Use all of our familiar inference techniques!

- Reducing FOL to PL is often impractical
 - ▣ Universal instantiation creates many sentences and propositions.
 - Remember that inference is exponential in number of propositions. (Why?)

 - ▣ Functions create infinitely large models
 - (Herbrand's theorem rescues us *a bit*)

 - ▣ Peano axioms create infinitely many propositions

Next time

- Inference within FOL (without reducing to PL)
 - ▣ Forward/backward chaining
 - ▣ Resolution