$$\frac{\forall v.\alpha}{Subst(\{v/g\},\alpha)}$$

MORE FIRST-ORDER LOGIC

EECS 492
February 15, 2011

Today's goals

- Practice understanding sentences
  - FOL -> English
- Practice translating sentences
  - English -> FOL
- Start FOL inference

Symbol Names

Compare:

- $\forall x.\ MyFriend(x) \Rightarrow SendBirthdayCard(x)$
- $\forall y.\ MyFriend(y) \Rightarrow SendBirthdayCard(y)$
- $\forall x.\ P0001(x) \Rightarrow P0002(x)$
- $\forall x.\ MyEnemy(x) \Rightarrow SendBirthdayCard(x)$

FOL to English

$\forall m,c.\ IsMotherOf(c,m) \Leftrightarrow IsFemale(m) \wedge IsParentOf(m,c)$

$\forall w,h.\ IsHusbandOf(w,h) \Leftrightarrow IsMale(h) \wedge IsSpouseOf(h,w)$

$\forall x.\ IsMale(x) \Leftrightarrow \neg IsFemale(x)$

$\forall p,c.\ IsParentOf(p,c) \Leftrightarrow IsChildOf(c,p)$

$\forall g,c.\ IsGrandparentOf(g,c) \Leftrightarrow \exists p.\ IsParentOf(g,p) \wedge IsParentOf(p,c)$

$\forall x,y.\ IsSiblingOf(x,y) \Leftrightarrow x \neq y \wedge \exists p.\ IsParentOf(p,x) \wedge IsParentOf(p,y)$

Your turn:

IsGrandChildOf, IsGreatGrandparentOf, IsBrotherInLawOf, IsFirstCousinOf, IsNthCousinOf,…

English to FOL

What's the "right" translation of the sentence "Not all students take both history and biology." ?

NotAllStudentsTakeBothHistoryAndBiology()

NotAllStudentsTakeBoth(History, Biology)

NotAllStudentsTake(History ∧ Biology)

NotAllStudentsTake(History) ∧ NotAllStudentsTake(Biology)

¬AllStudentsTakeBoth(History,Biology)

¬∀*x*. IsStudent(*x*) ⇒ TakesBoth(History,Biology)

¬∀*x*. IsStudent(*x*) ⇒ Takes(*x*,History) ∧ Takes(*x*,Biology)

(¬∀*x*. IsStudent(*x*) ⇒ Takes(*x*,History)) ∧ (¬∀*y*. Student(*y*) ⇒ Takes(*y*,Biology))

More History and Biology

- Not all students take both History and Biology.
- Only one student failed History.
- Only one student failed both History and Biology.
- The best score in History was better than the best score in Biology.

Temporal Sentences

- George is the President of the United States.
- The President of the United States has lived in the White House since 1803.
- The President of the United States has been sober since 1986.

FOL Inference: Reduction to PL

- Standard PL inference rules sound for FOL as well
  - E.g., *modus ponens*

$$\frac{\alpha \Rightarrow \beta, \alpha}{\beta}$$

IsFriend(Arnold)
IsFriend(Arnold) ⇒ShouldSendBirthdayCard(Arnold)

ShouldSendBirthdayCard(Arnold)

Universal Instantiation

- Replace a universally quantified variable with a ground term

$$\frac{\forall v.\alpha}{Subst(\{v/g\},\alpha)}$$

UI
∀x. IsFriend(x) ⇒ShouldSendBirthdayCard(x)
IsFriend(Arnold) ⇒ShouldSendBirthdayCard(Arnold)
MP
IsFriend(Arnold)
ShouldSendBirthdayCard(Arnold)

How should we handle existential quantification?

Existential Instantiation

- Replace an existentially quantified variable with a Skolem constant

$$\frac{\exists v.\alpha}{Subst(\{v/Sk\},\alpha)}$$

EI
∃x. IsFriend(x)
IsFriend(F0001)
UI, MP
∀x. IsFriend(x) ⇒ShouldSendBirthdayCard(x)
ShouldSendBirthdayCard(F0001)

FOL Inference

- We can now reduce FOL to PL inference:
  - Existentially instantiate everywhere.
  - Universally instantiate with respect to every object
  - Treat resulting terms as propositions
    - E.g., "IsFriend(FatherOf(Arnold))" is just a long name for a proposition.
- Uh oh!
  - Universal instantiation explodes if we have functions!
    - IsFriend(FatherOf(FatherOf(FatherOf(FatherOf(…

Herbrand's Theorem

- If a KB entails A, then there is a proof involving a *finite* subset of the propositionalized knowledge base.
  - I.e., any proof requires only a finite number of f(f(f(…)))'s
- What strategy does that suggest?

Semi-Decidability

- Does KB entail A?
  - Suppose we don't find a solution at depth 1…
  - Or depth 2.
  - Or depth 3.
  - Or depth 4.
  - …
- When can we state that KB does NOT entail A?
- Entailment of FOL is **semidecidable**: we can prove entailments, but can't disprove every non-entailed sentence.
  - Are there some non-entailed sentences that we *can* disprove?

Another problem with FOL → PL

- We can have an infinite number of propositions!
  - Proving statements about arithmetic
- Peano Axioms
  - NatNum(0)
  - ∀ n. NatNum(n) => NatNum(S(n))

Today's big idea

- We can reduce FOL to PL
  - Use all of our familiar inference techniques!
- Reducing FOL to PL is often impractical
  - Universal instantiation creates many sentences and propositions.
    - Remember that inference is exponential in number of propositions. (Why?)
  - Functions create infinitely large models
    - (Herbrand's theorem rescues us *a bit*)
  - Peano axioms create infinitely many propositions

Next time

- Inference within FOL (without reducing to PL)
  - Forward/backward chaining
  - Resolution