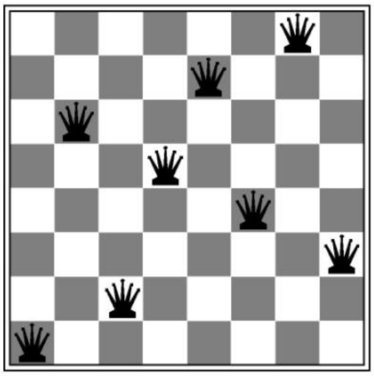
LOCAL SEARCH

EECS 492
January 20, 2011

Local Search

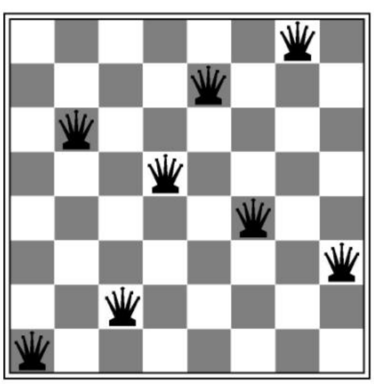
- Try to iteratively improve a small number of solutions
- Avoid space problems entirely: maintain only one a finite number of solution candidates
  - Perhaps only one!
- Repeatedly tweak those candidates in the hopes of arriving at a solution.
  - How do we tweak the solutions?

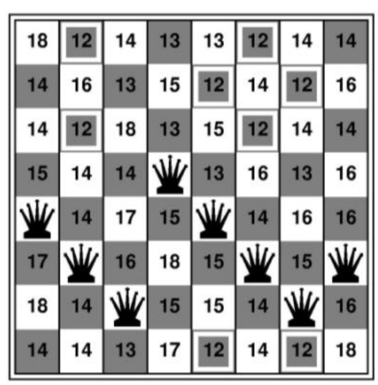
Example: 8 Queens



Find an arrangements of queens such that no queen attacks another

8 Queens Heuristic





$h = 1$ **a local minimum** $h = 17$

h: number of pairs of queens that attack each other

Local Beam Search

1. Randomly generate *k* initial states
2. Generate successors for each of them
3. If any successor is a goal, then return it and exit
4. Otherwise put all successors into queue, and sort queue.
5. Remove all but the *k* best nodes from the queue, and go to step 2

- How is this different than doing *k* random restarts?
- Can also have the stochastic variation, where the *k* nodes kept are chosen with some weighted probability based on heuristic value

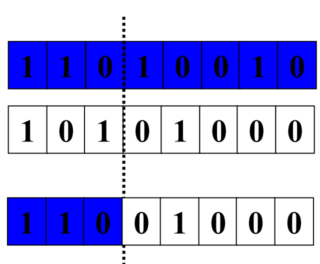
Genetic Algorithm

- Parallel hill climbing
- Candidate successors generated by crossover and mutation
- Actual successors then selected based on fitness

GA Steps

- Initialize population of size *N*
- Repeat *N* times:
  - Randomly select two “parents” from population, with probability proportional to fitness
  - Construct “child” by crossing over parents
  - Apply mutation with small probability

Crossing Over



- Randomly select crossover point.
- Child is same as parent1 up to crossover point, parent2 after that.

Genetic Algorithm: Your turn!

- You'll need a sheet of paper and a pencil
  - Write down four random numbers, x_1, x_2, x_3, and x_4.
  - Each number should be between [1, 9].
  - Seriously. They need to be random!
- You are our initial population!

Genetic Algorithm: Fitness

- Compute your fitness:

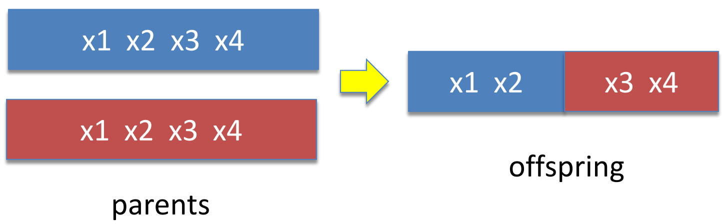
$$f = \left|6x_1^2x_2^2 + 18x_1^2x_4 - 70x_1^2 - 21x_2^2x_3 - 63x_3x_4 + \right.$$
$$\left. + 245x_3 + 24x_2^2 + 72x_4 - 280\right|$$

- (In our case, *small* fitnesses are good.)
- http://april.eecs.umich.edu/fitness.html

Genetic Algorithm: Reproduction

- Who has low fitnesses?

- Sexual reproduction (without mutation) by crossing (x1,x2) with (x3,x4)

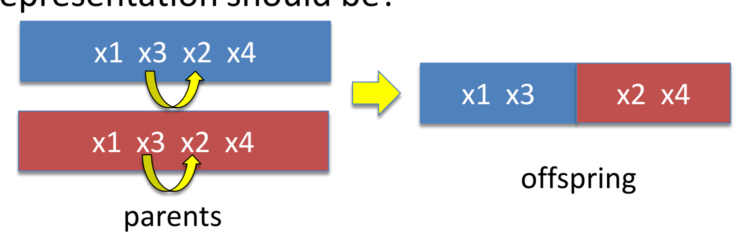


Changing the genetic representation

- Our fitness function can be factored like this:

$$\left|(2x_1^2 - 7x_3 + 8)(3x_2^2 + 9x_4 - 35)\right|$$

- What does this tell us about what our genetic representation should be?



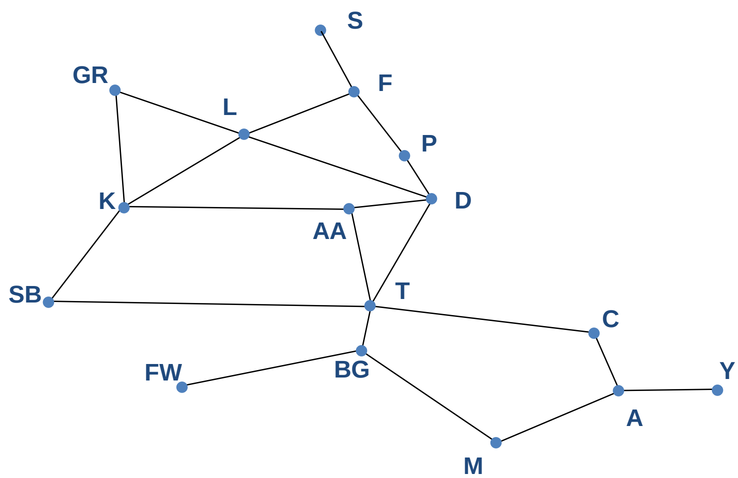
Adding Mutation

- We can randomly flip bits too…

Hill Climbing

- aka Gradient descent
- Requires heuristic *h* measuring quality of soln
- Algorithm:
  - Find all incremental modifications of candidate soln
  - Pick best one
  - Repeat

Example: Map Labeling



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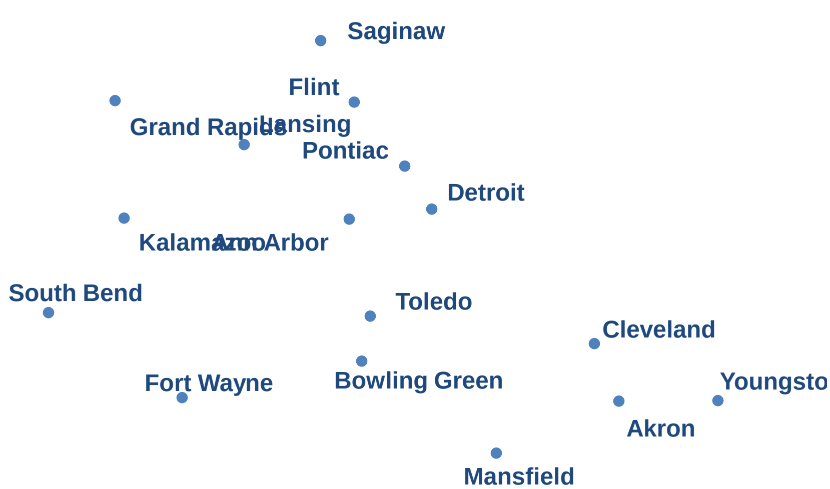

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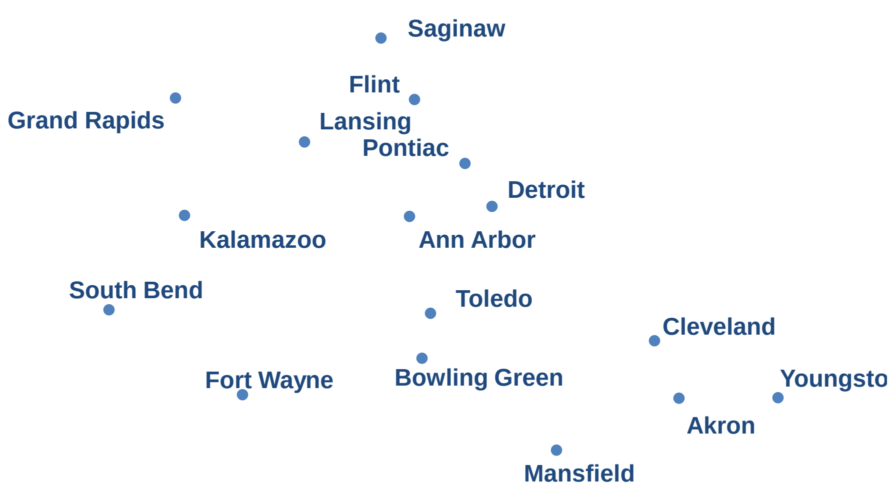

3SAT Example

$(P_1 \lor \neg P_2 \lor \neg P_3) \land (P_1 \lor \neg P_2 \lor \neg P_4) \land (P_1 \lor \neg P_3 \lor \neg P_4) \land$
$(\neg P_1 \lor P_2 \lor \neg P_3) \land (P_2 \lor \neg P_3 \lor \neg P_4) \land (\neg P_1 \lor P_2 \lor \neg P_4) \land$
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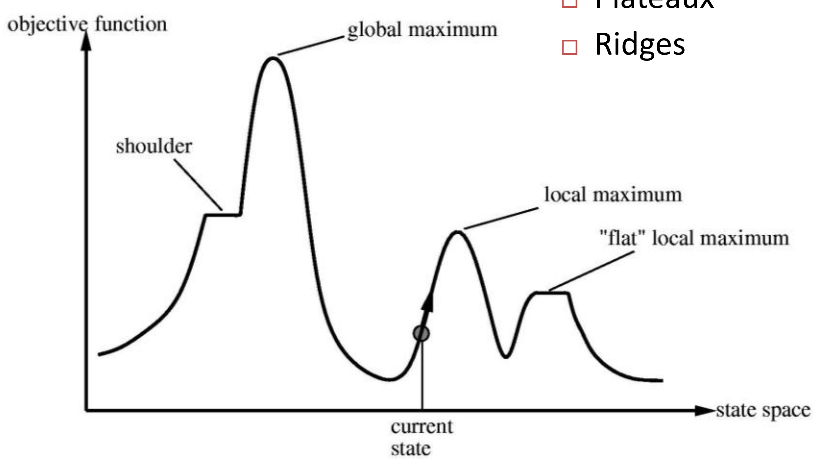
$(\neg P_1 \lor \neg P_2 \lor P_4) \land (\neg P_1 \lor \neg P_3 \lor P_4) \land (\neg P_2 \lor \neg P_3 \lor P_4) \land$
$(\neg P_1 \lor \neg P_2 \lor \neg P_3)$

Q: What's a good fitness function?

GSAT

procedure **GSAT(ϕ)**
    for **i := 1** to **Max-tries**
        T := random truth assignment
        for **j := 1** to **Max-flips**
            if **T satisfies ϕ** then return **T**
            else **Poss-flips := set of vars that increase satisfiability most**
                V := a random element of Poss-flips
                T := T with V's truth assignment flipped
        end
    end
    return **"no satisfying assignment found"**

Hill Climbing Terrain



- Local maxima
- Plateaux
- Ridges

Hill-Climbing: 2-d Ridge

2	1	1	1	1	1	2
1	4	3	1	3	4	1
1	3	6	5	6	3	1
1	1	5	10	5	1	1
1	3	6	5	6	3	1
1	4	3	1	3	4	1
2	1	1	1	1	1	2

Stochastic Variations

- Stochastic hill climbing
  - Select among positive steps at random
  - Probability proportional to steepness
- Random restarts
  - Repeat hill climbing from randomly chosen initial state
  - Return best local maximum found

- No clear answer on how often to restart from scratch versus trying to "repair" a current candidate that's stuck or making slow progress.

Simulated Annealing

- Hill climbing, but take worse-appearing steps with some probability
  - Generate random neighbor
    - If it is an improvement, accept;
    - else accept with probability < 1
  - probability decreases exponentially with the "badness" of the move, temperature
- Annealing: Decrease temperature gradually

- Stochastic Gradient Descent is similar
  - Useful for optimization with many simultaneous "soft" constraints
  - Temperature decreases as 1/T
    - Actually takes a *long* time for the temperature to get really small.

GA: Discussion

- Appealing analogy to natural selection with sexual reproduction
- Does it work?
  - Hard to characterize in general
  - Depends crucially on string rep'n of state
  - Intuition: GA maintains good "building blocks" in population
  - Not generally better than simpler stochastic local search methods

Assessing Local Search

- Key advantages
  - Very little memory
  - Can often find reasonable solutions in large or infinite (continuous) state spaces where other systematic approaches are unsuitable
- Usually incomplete and not optimal

Constraint satisfaction

- Next time…