UNINFORMED SEARCH

EECS492
January 13, 2011

General Tree Search

function Tree-search(*problem)*
returns a solution, or failure

fringe = new Set();
fringe.put(problem.initialState)

loop do
if *fringe*.isEmpty() **then return** failure
node 🡐 *fringe*.get()
if *problem*.isGoalState(*node*)
then return *node*;
fringe.putAll(*problem*.expand(*node*))

A problem is specified by:

- initialState
- actions/results 🡒 expand()
- isGoalState

Which node in the fringe does get() return?

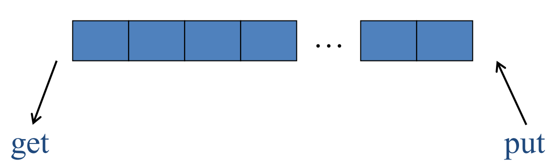
Measuring Search Performance

- Completeness
  - Is the algorithm guaranteed to find a solution if it exists?
- Optimality
  - Does the strategy find the minimum path cost solution?
- Time Complexity
  - How long does it take to find a solution?
- Space Complexity
  - How much memory is needed to perform the search?

Our search strategy will affect all of the above!

Breadth-First Search (BFS)

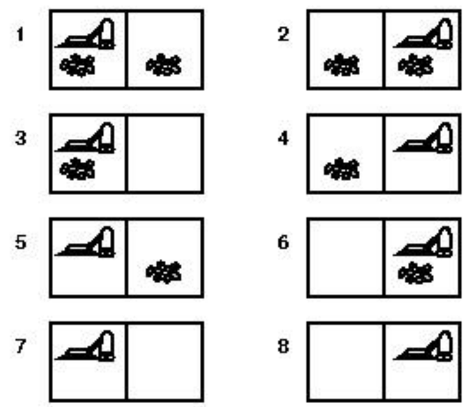
- General tree-search where queue is first-in-first-out (FIFO)
  - get operation returns oldest item on fringe



  - corresponds to a *level-order* traversal of search tree
- All nodes at level *d* expanded before any at *d*+1

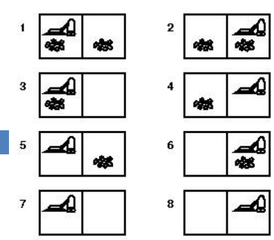
BFS in Vacuum World

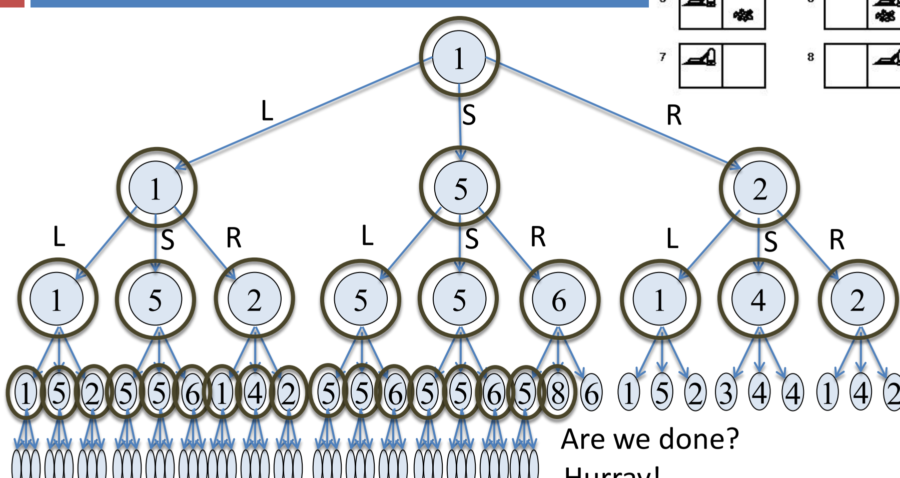
Actions:
L – move **Left**
S – **Suck** up the dirt
R – move **Right**



Starting in state 1, generate search tree.

Vacuum World BFS





Are we done?
Hurray!

BFS again

- What does BFS look like from the perspective of General Tree Search?

Search Performance Criteria: BFS

- Completeness: Guaranteed to find a solution if it exists?
  - YES
- Optimality: Does the strategy find the minimum path cost solution?
  - YES *if* uniform action cost
- Time Complexity: How long does it take to find a solution?
- Space Complexity: How much memory is needed to perform the search?

BFS Time Complexity

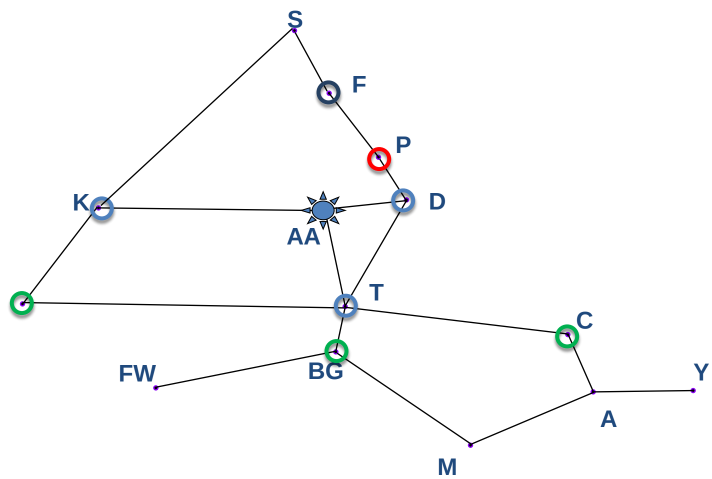
- Assume uniform search space
  - Each node has same # successors
  - branching factor, b
- d: Depth of shallowest solution
- BFS generates
  - complete search trees at depth $\leq d$
  - *Worst case:* b^{d+1} nodes at depth d+1
  - Total nodes:

$$b + b^2 + b^3 + \ldots + b^d + b^{d+1} = O(b^{d+1})$$

BFS Space Complexity

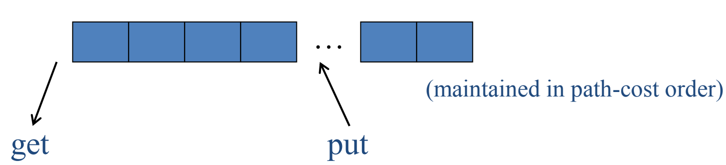
- Same as time complexity: $O(b^{d+1})$
- Must store entire fringe: all nodes at deepest level
- In typical computer configurations, will run out of space before running out of time

BFS and sub-optimal solutions



Uniform-Cost Search (UCS)

- General tree-search where queue is priority-first
  - get operation returns *least-cost* item on fringe



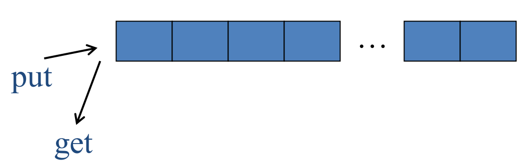
- All nodes at cost less than *c* expanded before any at cost *c*
  - Same as BFS if all actions have same cost, different otherwise

UCS Properties

- Complete?
  - Yes
- Time and space complexity
  - Uniform cost:
    - Same as BFS: $O(b^{d+1})$
  - In general, with minimal path cost C^* and minimal action cost ε:
    - $O(b^{C^*/\varepsilon + 1})$
- Optimal as long as cost monotonic along path
  - Guaranteed by each edge cost >= 0

Depth-First Search (DFS)

- General tree-search where queue is last-in-first-out (LIFO, aka *stack*)
  - get operation returns newest item on fringe

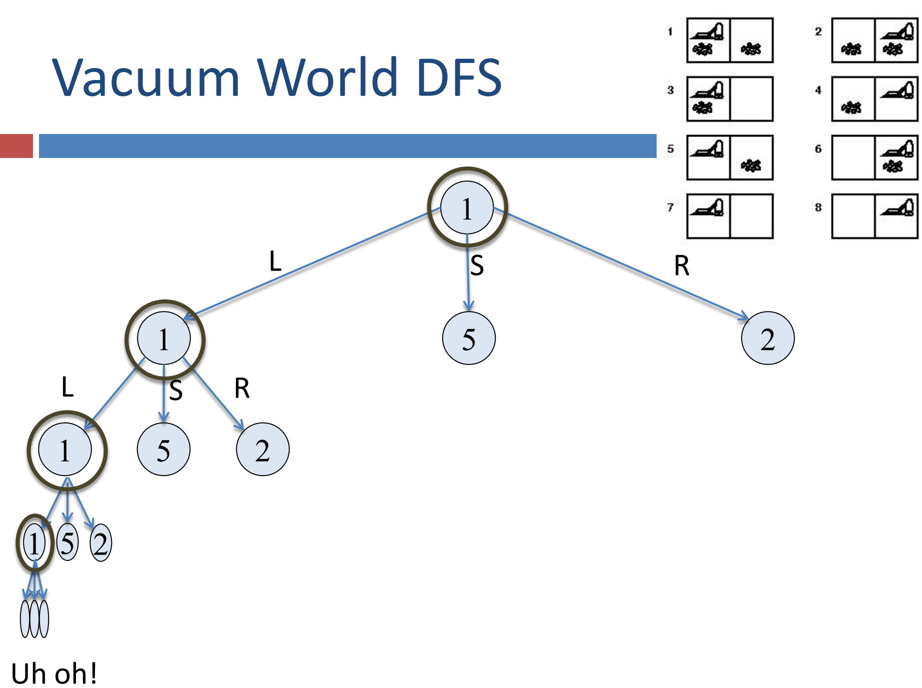


  - often implemented recursively using function-call stack
- Always expand deepest node on fringe

NOTE REGARDING NEXT SLIDE

- The nodes should really be expanded on the right side of the tree, so that the most recently added node is expanded.

Vacuum World DFS



Recursive implementation

- function search_recursive(state)
  - if (is_goal(state))
    - return state;
  - for (child : children(state))
    - v = search_recursive(child)
    - if (v != null)
      - return v;
  - return null;

Performance of Depth-First

- Completeness

  Only for finite depth trees, and so in general: No
- Optimality

  No
- Time Complexity

  $O(b^m)$; where m is the maximum depth of tree
- Space Complexity

  $O(b\ m)$

Depth-Limited Search

- Problem: unbounded trees in DFS
- Solution:
  - Predetermine a depth limit *L*
  - Run DFS, cut off search at depth *L*
- Complete iff exists solution within *L* steps
- Optimal?
- Time complexity: $O(b^L)$
- Space complexity: $O(bL)$

- **How do we choose *L*?**

Iterative Deepening DFS

function Iterative-deepening-search(*problem*)
  returns a solution, or failure

  loop for depth from 0 to infinity
    if Depth-limited-search(*problem*,*depth*) succeeds
      then return its result
    end loop
  return failure

IDS Properties

- Complete?
- Optimal?
- Time Complexity?
- Space Complexity?

IDS Complexity

- Time is same as running DLS for depth = 1,…,*d*

$$\sum_{l=1}^{d} O(b^l) = O(b) + O(b^2) + \cdots + O(b^d) = O(b^d)$$

- Space is same as DLS for depth *d*: O(*bd*)

Analysis Summary

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening
Complete?	Yes	Yes	No	No	Yes
Optimal?	Yes	Yes	No	No	Yes
Time complexity	$O(b^{d+1})$	$O(b^{C^*/\varepsilon+1})$	$O(b^m)$	$O(b^L)$	$O(b^d)$
Space compexity	$O(b^{d+1})$	$O(b^{C^*/\varepsilon+1})$	O(bm)	O(bL)	O(bd)

Exponential in depth!

Linear in depth!

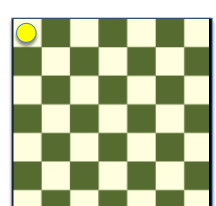
Your turn!

- Show that:

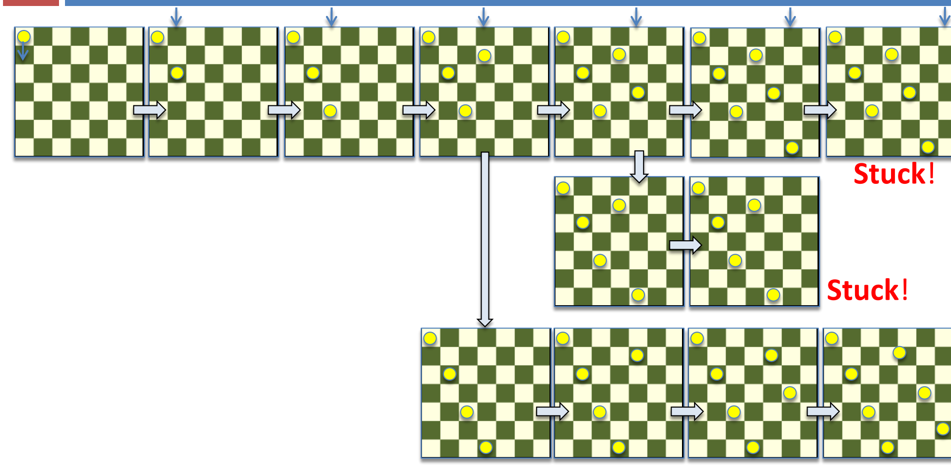
$$b + b^2 + b^3 + ... + b^n = O(b^n)$$

  - (by finding a polynomial of order b^n with constant coefficients that is greater than the sum)

- 7 Queens:
  - Goal: Place 7 queens on 7x7 chess board so that no two attack each other
  - Formulate the problem carefully
    - What state space?
    - What actions do you consider at each step?
  - Which search strategy to use?
    - Find a solution using your strategy.



7 queens, DFS



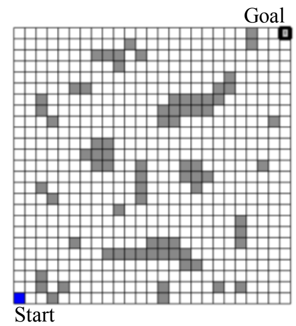
The importance of problem formulation

- Problem formulation has huge impact on complexity
  - State space
  - Actions

- Consider 7-queens problem:
  - Naïve state space: branching factor of roughly ~49
  - One queen per column: branching factor of 7.

Path Planning Example

- Consider an agent trying to find *the best* route from one place to another.
  - Actions = {N, S, E, W}
  - DFS is out. (Why?)



- The shortest path is 50 moves.
  - Complexity of BFS/IDS?
- How many distinct states are there?
  - $25^2 << 4^{50}$ (by a factor of 10^{27}!)
  - What are we doing wrong?
    - Repeated states!

Avoiding repeated states

- Idea: don't re-expand nodes that we've already expanded.
  - Closed list: set of all states previously visited
  - Memory usage?

General Graph Search

```
function Tree-search(problem)
    returns a solution, or failure

    closed = new Set();
    closed.add(problem.initialState);

    fringe = new Set();
    fringe.put(problem.initialState)

    loop do
        if fringe.isEmpty() then return failure
        node ← fringe.get()
        if problem.isGoalState(node)
            then return node;

        for each child in problem.expand(node)
            if !set.contains(child)
                closed.add(child)
                fringe.put(child)
```

General Graph Search: Analysis

- Time complexity?
  - O(|V| + |E|)
- Memory complexity?
  - O(|V|)
- Optimality?
  - BFS/IDS?
  - DFS?
- What about DFS's infinite loop problem?
  - Fixed for finite worlds!
- Can we guarantee optimality, regardless of the policy?
  - For finite worlds, if we replace old bad paths with new good paths rather than discarding the new good paths.
  - Some bookkeeping…

Graph Search: Summary

- Graph Search
  - Maintains list of states already visited
  - Terminates searches that revisit the same states.
- When do we want to use graph search?
  - When repeated states are likely
  - We can afford the memory

Next Time: Informed Search

- Employ additional information about node states in deciding which to expand
- Questions:
  - What criteria?
  - How to exploit?
  - Properties?