Suppose robot is at $[0,0]$ (or equivalently, consider motion in robot-relative coordinates.)

\[ \phi = \frac{dL}{r_L} = \frac{dR}{r_R} = \frac{dc}{r_c} \]

\[ r_2 = r_1 + b \quad \text{baseline dist. betw. wheels} \]

\[ \implies \phi = \frac{dr - dc}{b} \quad \implies \text{this is also change in heading} \]

\[
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta \theta
\end{bmatrix} = \begin{bmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{bmatrix} \begin{bmatrix}
0 \\
0
\end{bmatrix} + \begin{bmatrix}
0 \\
0
\end{bmatrix} = \begin{bmatrix}
r_c \sin \phi \\
r_c - r_c \cos \phi
\end{bmatrix}
\]

but \[ r_c = \frac{dt + dr}{2\phi} \quad \text{and small angles, } \sin \phi \approx \phi \text{ and } \cos \phi \approx 1 \]

\[ \text{so} \]

\[
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta \theta
\end{bmatrix} = \begin{bmatrix}
r_c \phi \\
0 \\
\phi
\end{bmatrix} = \begin{bmatrix}
\frac{dt + dr}{2} \\
0 \\
\phi
\end{bmatrix}
\]
Suppose \( d_L, d_R \) contaminated by white noise,
\[
d_L = d_L^* + \omega_L \quad \omega_L \sim N(0, \sigma_L^2)
\]
\[
d_R = d_R^* + \omega_R \quad \omega_R \sim N(0, \sigma_R^2)
\]

\[
\Sigma_{xy,\theta} = \int_{dL} \int_{dR} \Sigma_{dL, dR}
\]

**What does our new position look like if \( x_0 = \begin{bmatrix} x_0 \\ y_0 \\ \theta_0 \end{bmatrix} \)?**

\[
x_1 = \begin{bmatrix} x_0 \\ y_0 \\ \theta_0 \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta \theta \end{bmatrix}
\]

\[
E[x_1] = E[x_0] + \Sigma_{\Delta x, \Delta y, \Delta \theta}
\]

\[
\Sigma_{x_1} = \Sigma_{x_0} + \Sigma_{\Delta x, \Delta y, \Delta \theta}
\]

does this make sense?

A problem to try at home:

bicycle model